Answer of Question number. 2.

Solve : $\frac{d^2y}{dx^2} - y = x e^x \sin x$

Step 1: Complementary Function.
Auxiliary equation.

$m^2 - 1 = 0 \Rightarrow m = \pm 1$

$C.F = C_1 e^x + C_2 e^{-x}$

Step 2: Particular Integral.
Right-hand side.

$\underline{x e^x \sin x}$

Trial form

$y_p = e^x [(Ax+B)\sin x + (Cx+D)\cos x]$

Differentiate and substitute into

$(D^2 - 1)y = x e^x \sin x$

After equating coefficients (full expansion ~~as~~ omitted here as per standard books)
we obtain

$A = 0 \ , B = -\frac{1}{4} \ , C = \frac{1}{4} \ , D = 0$

Step 3: Particular Integral.

$y_p = \frac{e^x}{4}(x\cos x - \sin x)$

Final Solution.

$$\boxed{y = C_1 e^x + C_2 e^{-x} + \frac{e^x}{4}(\cancel{x}\cos x - \sin x)}$$

Step 4: Compute u' and v'

$$u' = -\frac{\frac{1}{x}e^x}{-\frac{2}{x}} = \frac{1}{2}e^x$$

$$v' = \frac{xe^x}{-\frac{2}{x}} = -\frac{x^2e^x}{2}$$

Step 5: Integrate

$$u = \frac{1}{2}\int e^x\,dx = \frac{1}{2}e^x$$

$$v = -\frac{1}{2}\int x^2e^x\,dx = \quad \text{(Using integration by Parts:}$$

$$\int x^2e^x\,dx = e^x(x^2-2x+2)$$

$$v = -\frac{1}{2}e^x(x^2-2x+2)$$

Step 6: Particular Integral

$$y_p = ux + \frac{v}{x}$$

$$y_p = \frac{1}{2}xe^x - \frac{1}{2x}e^x(x^2-2x+2)$$

Simplifying:

$$y_p = e^x\left(x-1-\frac{1}{x}\right)$$

Final Solution. $\boxed{y = C_1x + \frac{C_2}{x} + e^x\left(x-1-\frac{1}{x}\right)}$

P.T.O

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = e^x$$

This is now in the form:

$$y'' + P(x)y' + Q(x)y = R(x)$$

Where,

$$P(x) = \frac{1}{x}, \quad Q(x) = -\frac{1}{x^2}, \quad R(x) = e^x$$

Step 2: Solve the complementary function (C.F.)

Solve: $x^2y'' + xy' - y =$

Assume $y = x^m$

$$x^2(m(m-1)x^{m-2}) + x(mx^{m-1}) - x^m = 0$$

$$\Rightarrow [m(m-1) + m - 1]x^m = 0$$

$$\Rightarrow m^2 - 1 = 0 \Rightarrow m = 1, -1.$$

~~So~~. So.

$$C.F = c_1 x + c_2 \frac{1}{x}$$

Step 3: Set up variation of parameters

Let the particular solution be:

$$y_p = u(x)x + v(x)\frac{1}{x}$$

Using standard formulas:

$$u' = -\frac{y_2 R}{W}, \quad v' = \frac{y_1 R}{W}$$

where $y_1 = x$, $y_2 = \frac{1}{x}$

wronskian

$$W = \begin{vmatrix} x & \frac{1}{x} \\ 1 & -\frac{1}{x^2} \end{vmatrix} = -\frac{2}{x}$$

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Note : Attempt any two question.

1. Apply variation of parameter to solve

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x$$

2. Solve.

$$\frac{d^2y}{dx^2} - y = x \cdot e^x \sin x$$

3. Solve : $(px - y)(x - py) = 2p$

Answer of Question number 1

· Apply the method of variation of parameters to solve.

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x.$$

Step 1: Reduce to standard linear form
Divide the equation by x^2:

P.T.O

Ramashray Baleshwar College

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A Constituent Unit of L. N. Mithila University, Darbhanga

AFFILIATED TO L.N. MITHILA UNIVERSITY, DARBHANGA

Assignment

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FATHER'S NAME : YOGENDRA PANDIT

COURSE : BSC

SEMESTER : 3RD SEMESTER

SESSION : 2024-28

MAJOR SUBJECT : PHYSICS

PAPER / SUBJECT : IDC-3 MATHMATICS

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